



12 Pages

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
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(see note 2 below)

SURNAME: NUNNERY

FORENAME(S): SIÂN

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2	Title of Examination Paper	TCA 2
3	Module Code	PC.EE4000 PC.P04000
4	Date and Time of Examination	19.05.2021
5	Examination Venue	pembroke college
6	Seat Number	
7	Number of Answer Books handed in	

List the numbers of the questions you have attempted	Office Use
1	22
2	7
3	16
4	18
5	9
6	10
7	8
Office Use	90 ✓

Second Marker
SD 711
25/05/21

Q1a) $y = \frac{1}{3x^4} - 2x^5 - \sqrt{x}$

$$y = \frac{1x^{-4}}{3} - 2x^5 - x^{1/2}$$

$$\frac{dy}{dx} = \frac{-4x^{-5}}{3} - 10x^4 - \frac{1x^{-1/2}}{2}$$

$$\frac{dy}{dx} = -\frac{4}{3x^5} - 10x^4 - \frac{1}{2\sqrt{x}}$$

4 ✓

b) $y = \frac{5\cos 2x}{x^4}$

$$u = 5\cos 2x \quad v = x^4$$

$$\frac{du}{dx} = -10\sin 2x \quad \frac{dv}{dx} = 4x^3$$

$$\frac{dy}{dx} = \frac{x^4 \times -10\sin 2x - 5\cos 2x \times 4x^3}{(x^4)^2}$$

$$\frac{dy}{dx} = \frac{-10x^4 \sin 2x - 20x^3 \cos 2x}{(x^4)^2}$$

$$\frac{dy}{dx} = \frac{10x^3 (-x \sin 2x - 2 \cos 2x)}{2^8}$$

6 ✓

c) $y = 2\cos(4x^2)$

$$y = 2\cos(u)$$

$$\frac{dy}{du} = -2\sin(u)$$

$$u = 4x^2$$

$$\frac{du}{dx} = 8x$$

$$\frac{dy}{dx} = -2\sin(u) \times 8x$$

$$= -16\sin(4x^2)$$

6 ✓

d) $y = 5x^2 e^{4x}$

$$u = 5x^2 \quad v = e^{4x}$$

$$\frac{du}{dx} = 10x \quad \frac{dv}{dx} = 4e^{4x}$$

$$\frac{dy}{dx} = 5x^2 \times 4e^{4x} + e^{4x} \times 10x$$

$$\frac{dy}{dx} = 20x^2 e^{4x} + 10x e^{4x}$$

$$\frac{dy}{dx} = 10x e^{4x} (2x + 1)$$

⑥ ✓

Q2 $v = 10(2 - 3e^{-4t})$ $t = 0$

$$v = 20 - 30e^{-4t}$$

$$\frac{dv}{dt} = 120e^{-4t}$$

at $t = 0$

$$= 120e^{-4 \times 0}$$

$$= \underline{\underline{120 \text{ v/s}}}$$

⑦ ✓

Q3a) $\int_3^4 (20 + 5x^2 - 2x^3) dx$

$$= \frac{20x^{0+1}}{0+1} + \frac{5x^{2+1}}{2+1} - \frac{2x^{3+1}}{3+1}$$

$$= 20x + \frac{5x^3}{3} - \frac{2x^4}{4}$$

$$= \left[20x + \frac{5x^3}{3} - \frac{2x^4}{4} \right]_3^4$$

upper limit

$$x = 4$$

$$= 20(4) + \frac{5(4)^3}{3} - \frac{(4)^4}{2} = \frac{176}{3}$$

lower limit

$$x = 3$$

$$= 20(3) + \frac{5(3)^3}{3} - \frac{(3)^4}{2} = \frac{129}{2}$$

$$\text{Area} = \frac{176}{3} - \frac{129}{2} = \underline{\underline{-5.83 \text{ units}^2}}$$

⑧ ✓

$$b) \int_0^{\pi} 3 \sin \frac{x}{2} - \cos \frac{x}{4} dx$$

~~$$= -3 \times \frac{1}{2} \cos \frac{x}{2} - \frac{1}{4} \sin \frac{x}{4}$$~~

~~$$= -\frac{3}{2} \cos \frac{x}{2} - \frac{1}{4} \sin \frac{x}{4}$$~~

~~$$= \left[-\frac{6 \cos \frac{x}{2}}{2} - \frac{1 \sin \frac{x}{4}}{4} \right]_0^{\pi}$$~~

$$= -\frac{3}{2} \cos \frac{x}{2} - \frac{1}{4} \sin \frac{x}{4}$$

$$= \left[-\frac{6 \cos \frac{x}{2}}{2} - \frac{4 \sin \frac{x}{4}}{4} \right]_0^{\pi}$$

upper limit

$$x = \pi = -\frac{6 \cos \frac{\pi}{2}}{2} - \frac{4 \sin \frac{\pi}{4}}{4} = -2\sqrt{2}$$

or

lower limit

$$x = 0 = -\frac{6 \cos 0}{2} - \frac{4 \sin 0}{4} = -6$$

$$\text{Area} = -2\sqrt{2} - -6 = \underline{\underline{3.17 \text{ units}^2}}$$

8 ✓

$$Q4 \quad \frac{2x+1}{4x(x+2)} = \frac{A}{4x} + \frac{B}{x+2} = \frac{A(x+2) + B(4x)}{4x(x+2)}$$

$$2x+1 = A(x+2) + B(4x) \quad \checkmark$$

$$x=0 \quad 2(0)+1 = A(0+2) + B(4 \times 0)$$

$$1 = 2A$$

$$\frac{1}{2} = A \quad \checkmark$$

$$x=-2 \quad 2(-2)+1 = A(-2+2) + B(4 \times -2)$$

$$-3 = -8B$$

$$\frac{3}{8} = B \quad \checkmark$$

$$\int_1^3 \frac{2x+1}{4x(x+2)} dx = \int_1^3 \left(\frac{1/2}{4x} + \frac{3/8}{x+2} \right) dx \quad \checkmark$$

$$= \frac{1}{2} \times \frac{1}{4} \ln|4x| + \frac{3}{8} \times \frac{1}{1} \ln|x+2| \quad \checkmark$$

$$= \left[\frac{1}{8} \ln|4x| + \frac{3}{8} \ln|x+2| \right]_1^3 \quad \checkmark$$

upper limit

$$x=3 \quad = \frac{1}{8} \ln|4 \times 3| + \frac{3}{8} \ln|3+2|$$

$$= 0.91415 \quad \checkmark$$

lower limit

$$x=1 \quad = \frac{1}{8} \ln|4 \times 1| + \frac{3}{8} \ln|1+2|$$

$$= \cancel{0.0} 0.58527 \quad \checkmark$$

$$\text{Area} = 0.91415 - 0.58527 = \underline{\underline{0.3289 \text{ units}^2}} \quad \checkmark \quad (4dp)$$

18 ✓

Q5. $\int_1^3 \frac{2x+1}{4x(x+2)} dx$

$$h = \frac{3-1}{4} = \frac{1}{2} \quad \checkmark$$

	1	1.5	2	2.5	3	✓
$\frac{2x+1}{4x(x+2)}$	$\frac{1}{4}$	$\frac{4}{21}$	$\frac{5}{32}$	$\frac{2}{15}$	$\frac{7}{60}$	✓
	y_0	y_1	y_2	y_3	y_4	

Simpson's Rule.

$$= \frac{1}{3} \times \frac{1}{2} \left[\left(\frac{1}{4} + \frac{7}{60} \right) + 4 \left(\frac{4}{21} + \frac{2}{15} \right) + 2 \left(\frac{5}{32} \right) \right]$$

$$= \underline{\underline{0.3246}} \quad (4dp)$$

$$\% \text{ error} = \frac{\text{actual} - \text{approx}}{\text{actual}} \times 100$$

$$= \frac{0.3289 - 0.3246}{0.3289} \times 100$$

$$= \underline{\underline{1.31\%}}$$

(4) ✓

$$\sum f = 60$$

Q6.	4	3	19	24	10	4	
x	1	3	5	7	9		✓
fx	3	57	120	70	36		$= 286$ ✓
$f(x-\bar{x})^2$	$\frac{26508}{625}$	$\frac{36784}{625}$	$\frac{864}{625}$	$\frac{6272}{125}$	$\frac{44944}{625}$		$= 224.736$

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{286}{60} = \frac{143}{30} = 4.76 \text{ rounding!}$$

$$s.d = \sigma = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = 1.935 \text{ ✓}$$

(10) ✓

Q7. $P(A) = 0.77$ $P(B) = 0.81$

$$\begin{aligned} a) P(\text{connection}) &= P(A) \times P(B) \\ &= 0.77 \times 0.81 \\ &= \underline{\underline{0.6237}} \text{ ✓} \end{aligned}$$

$$\begin{aligned} b) P(\bar{A}) &= 1 - 0.77 = 0.23 \\ P(\bar{B}) &= 1 - 0.81 = 0.19 \end{aligned}$$

$$\begin{aligned} P(\text{connection}) &= 0.23 \times 0.19 \\ &= \underline{\underline{0.0437}} \text{ ✓} \end{aligned}$$

$$\begin{aligned} P(\text{connection}) &= 1 - 0.437 \\ &= \underline{\underline{0.9563}} \text{ ✓} \end{aligned}$$

(8) ✓